HE 215 : Nuclear & Particle Physics Course

Dr. Jyothsna Rani Komaragiri

Centre for High Energy Physics, IISc

Webpage: `http://chep.iisc.ac.in/Personnel/jyothsna.html`

e-mail:
jyothsna@iisc.ac.in,
jyothsna.komaragiri@gmail.com

November 2018 Lectures

Outline I

- Electrodynamics & Chromodynamics of Quarks
  - Deep Inelastic Scattering
  - The Parton Model and Bjorken Scaling

Electrodynamics & Chromodynamics of Quarks

This is chapter 8 in Griffiths.

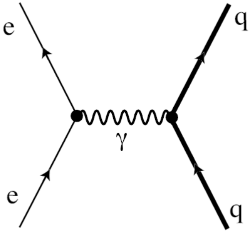
We will now get into chapter 8. That allows us to look at QED interactions of quarks.

We will do deep-inelastic scattering, the parton model, etc.
...following “First edition of Griffiths”
We will also learn Feynman rules for QCD.

In some ways this will seem like a digression since we then come back to weak interactions.

Electrodynamics & Chromodynamics of Quarks

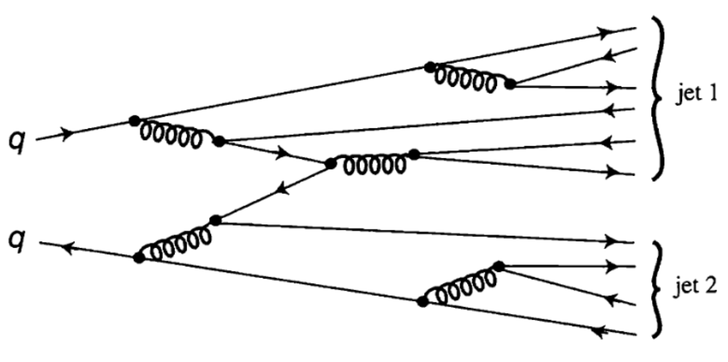
We have been doing QED Feynman diagrams in which the only particles were leptons and photons. However, quarks are charged too. What about their diagrams?

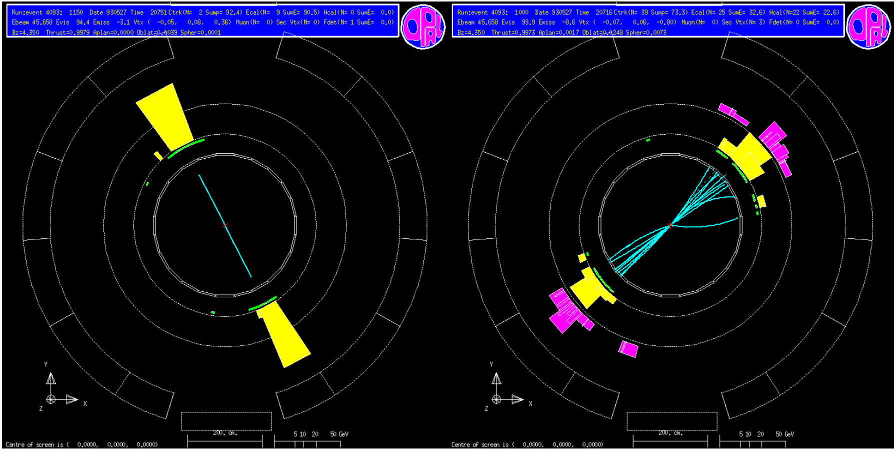


We will also see later that replacing the photon in the above diagram with a Z boson is almost the same thing....anywhere you can put a photon you could put a Z instead.

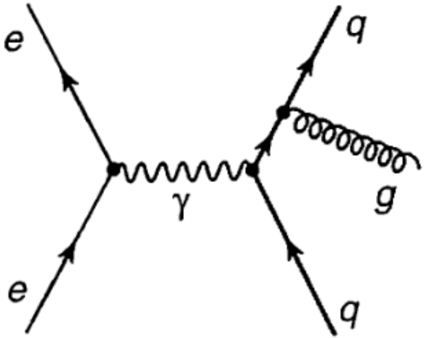
When quarks are produced we do not see two back-to-back particles in our detector, instead we see jets:

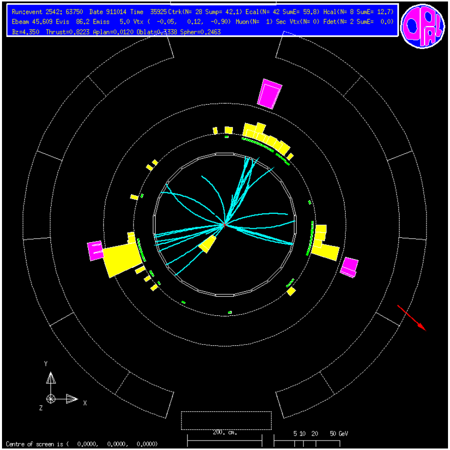
Hadronization and jet formation



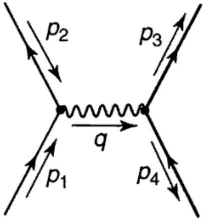

Run:event 4093: 1150 Date 930527 Time 20751 Ctrk(N= 2 Sump= 92.4) Ecal(N= 9 SumE= 90.5) Hcal(N= 0 SumE= 0.0)
Ebeam 45.658 Evis 94.4 Emiss -3.1 Vtx (-0.05, 0.08, 0.36) Muon(N= 0) Sec Vtx(N= 0) Fdet(N= 1 SumE= 0.0)
Bz=4.350 Thrust=0.9979 Aplan=0.0000 Oblat=0.0039 Spher=0.0001
Y
Z
X
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)
Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
Z
X
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

Hadron Production in e^+e^- Scattering





Hadron Production in e^+e^- Scattering

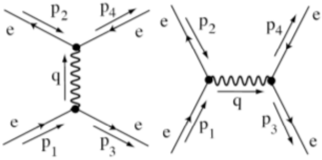


This diagram is calculated just like $e^+e^- \to \gamma \to e^+e^-$ with amplitude

$$\mathcal{M} = \frac{Qg_e^2}{(p_1 + p_2)^2}[\bar{v}(2)\gamma^\mu u(1)][\bar{u}(3)\gamma_\mu v(4)]$$

where Q is the quark charge in units of e.

Electron-Positron Scattering



The second diagram gives

$$(2\pi)^4 \int [\bar{u}(3)(ig_e\gamma^\mu)v(4)]\frac{-ig_{\mu\nu}}{q^2}[\bar{v}(2)(ig_e\gamma^\nu)u(1)]$$
$$\times\, \delta^4(q-p_3-p_4)\delta^4(p_1+p_2-q)d^4q$$

Which gives the amplitude

$$\mathcal{M}_2 = -\frac{g_e^2}{(p_1+p_2)^2}[\bar{u}(3)\gamma^\mu v(4)][\bar{v}(2)\gamma_\mu u(1)]$$

Hadron Production in e^+e^- Scattering

Using Casimir's Trick and the trace theorems (see your text) we can obtain:

$$\langle|\mathcal{M}|^2\rangle = Q^2 g_e^4 \left\{1 + \left(\frac{mc^2}{E}\right)^2 + \left(\frac{Mc^2}{E}\right)^2 + \left[1 - \left(\frac{mc^2}{E}\right)^2\right]\left[1 - \left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right\}$$

We could plug this in, integrate and get the total cross section and get

$$\sigma = \frac{\pi Q^2}{3}\left(\frac{\hbar c\alpha}{E}\right)^2 \sqrt{\frac{1-(Mc^2/E)^2}{1-(mc^2/E)^2}}\left[1 + \frac{1}{2}\left(\frac{Mc^2}{E}\right)^2\right]\left[1 + \frac{1}{2}\left(\frac{mc^2}{E}\right)^2\right]$$

Notice the threshold effect at $E = Mc^2$.

Hadron Production in e^+e^- Scattering

This simplifies considerably if $E > Mc^2 \gg mc^2$

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar Q c \alpha}{E}\right)^2$$

Thresholds appear in the cross section at the masses of the quark pairs we need to create in the process.
This can be displayed in a ratio:

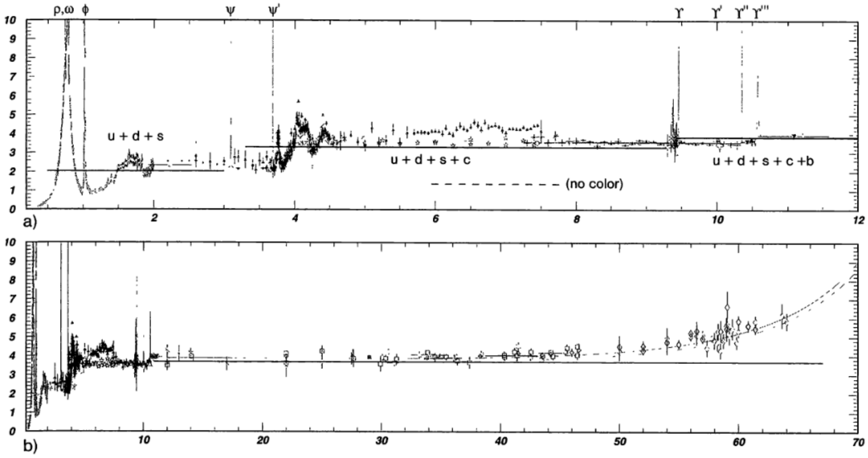
$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

From the simplified cross section expression we could derive

$$R(E) = 3\sum Q_i^2$$

Note: The factor of **3** is due to the fact that there are three colors for each quark flavor.

Hadron Production in e^+e^- Scattering



R=2 when u, d, s contribute, R=3.33 between c & b threshold, R=3.67 at b threshold and t should produce a jump to R=5

Hadron Production in e^+e^- Scattering

The experimental results are shown in Figure 8.4. The agreement between theory and experiment is pretty good, especially at high energy. But you may well ask why it is not *perfect*. Apart from the approximation in going from Equation 8.5 to Equation 8.6 (which artificially sharpens the corners at each threshold), and the neglect of the tau, we have made a fundamental oversimplification in assuming that we could treat the process as a sequence of two independent operations: $e^+e^- \to q\bar{q}$ (QED) followed by $q\bar{q} \to$ hadrons (QCD). In point of fact, the quarks produced in the first step are *not* free particles, obeying the Dirac equation; rather, they are *virtual* particles, on their way to a second interaction. This is particularly critical when the energy is right for formation of a bound state ($\phi = s\bar{s}$, $\psi = c\bar{c}$, $\Upsilon = b\bar{b}$); in the vicinity of such a 'resonance', the interaction of the two quarks can scarcely be ignored. Hence the sharp spikes in the graph, which typically occur just below each threshold. Finally, above about 50 GeV, the graph starts to rise toward the Z^0 peak, at 91 GeV.

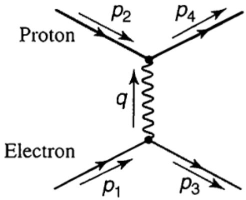
More on Protons, Quarks and QCD

The next bit is full of form factors, structure functions, etc. In other words, everything gets complicated and we frequently hide our lack of understanding.

However, if you want to understand electron-proton or proton-proton interactions then we need to understand this part of chapter 8 at least.

Elastic Electron-Proton Scattering

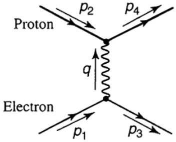
Our best probe of the internal structure of the proton is e-p scattering. We have already studied e-μ scattering in some detail in chapter 7. If protons were just heavy muons then there would be nothing new to learn here.We'd do:



with

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4}{4(p_1-p_3)^4}[4(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu + (m^2 - p_1\cdot p_3)g^{\mu\nu})] \times [4(p_{2\mu}p_{4\nu} + p_{4\mu}p_{2\nu} + (M^2 - p_2\cdot p_4)g_{\mu\nu})]$$

Elastic Electron-Proton Scattering



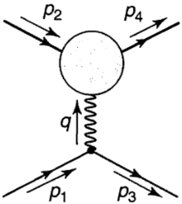
$$
\begin{aligned}
\langle|\mathcal{M}|^2\rangle &= \frac{g_e^4}{4(p_1 - p_3)^4}[4(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu + (m^2 - p_1 \cdot p_3)g^{\mu\nu})] \\
&\times \; [4(p_{2\mu} p_{4\nu} + p_{4\mu} p_{2\nu} + (M^2 - p_2 \cdot p_4)g_{\mu\nu})] \\
&= \frac{g_e^4}{q^4} L^{\mu\nu}_{electron} L_{\mu\nu proton}
\end{aligned}
$$

with

$$L^{\mu\nu}_{electron} = 2(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu + ((mc)^2 - p_1 \cdot p_3)g^{\mu\nu})$$

Elastic Electron-Proton Scattering

Unfortunately, what we have is

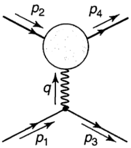


As usual, the blob means we are hiding something we don't really understand. We then write

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4}{q^4} L^{\mu\nu}_{electron} K_{\mu\nu proton}$$

where $K_{\mu\nu}$ is unknown quantity describing the photon-proton vertex. Note that the electron-photon coupling and the photon propagator is unaffected.

Elastic Electron-Proton Scattering



OK, now we need to figure out $K_{\mu\nu}$. We already know that it is a second-rank tensor and that it can only depend on p_2, p_4 and q. However, we also know that $q = p_4 - p_2$, so there are only 2 independent variables this tensor can depend on. We choose two: q and $p_2 = p$. Thus, the possible terms are

$$g^{\mu\nu},\ p^\mu p^\nu,\ q^\mu q^\nu,\ (p^\mu q^\nu + p^\nu q^\mu),\ (p^\mu q^\nu - p^\nu q^\mu)$$

and the most general for of K we can write is

$$K^{\mu\nu}_{proton} = -K_1 g^{\mu\nu} + \frac{K_2}{(Mc)^2} p^\mu p^\nu + \frac{K_4}{(Mc)^2} q^\mu q^\nu + \frac{K_5}{(Mc)^2}(p^\mu q^\nu + p^\nu q^\mu)$$

Elastic Electron-Proton Scattering

$$K^{\mu\nu}_{proton} = -K_1 g^{\mu\nu} + \frac{K_2}{(Mc)^2} p^\mu p^\nu + \frac{K_4}{(Mc)^2} q^\mu q^\nu + \frac{K_5}{(Mc)^2} (p^\mu q^\nu + p^\nu q^\mu)$$

K_1, K_2, K_4, K_5 are called "form factors" and are unknown. These form factors depend on q^2, the only scalar variable we have ($p^2 = M^2$ and $p \cdot q = -q^2/2$ which comes from squaring $p_4 = p_2 + q$).
The Ward identity tells us

$$q_\mu K^{\mu\nu} = 0$$

This constraint allows us to write $K^{\mu\nu}$ in terms of only two independent form factors

$$K^{\mu\nu}_{proton} = K_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{K_2}{(Mc)^2} \left(p^\mu + \frac{1}{2} q^\mu \right) \left(p^\nu + \frac{1}{2} q^\nu \right)$$

Our goal is to measure these two form factors experimentally and calculate them theoretically.

Elastic Electron-Proton Scattering

Experimentally we can see that the two form factors are directly related to the cross section. In the lab frame we have (see your text):

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4 c^2}{4EE' \sin^4(\theta/2)}\left(2K_1 \sin^2\frac{\theta}{2} + K_2 \cos^2\frac{\theta}{2}\right)$$

The outgoing electron energy is E' and it is completely kinematically determined by E and θ

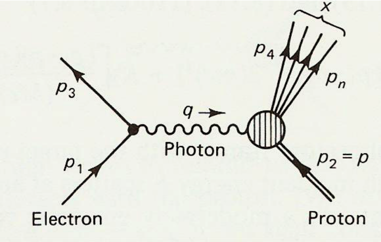
$$E' = \frac{E}{1 + (2E/Mc^2)\sin^2(\theta/2)}$$

Leading to the Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{4ME\sin^2(\theta/2)}\right)^2 \frac{E'}{E}\left[2K_1 \sin^2(\theta/2) + K_2 \cos^2(\theta/2)\right]$$

Inelastic Electron-Proton Scattering

If the probe electron is sufficiently energetic we move from the elastic scattering regime to the inelastic regime. In elastic scattering the proton is always just a proton, in inelastic scattering any number of things could come out ($e + p \rightarrow e + X$) (time up):



Again, the blob hides some interactions we don't fully understand. We write

$$\langle |\mathcal{M}|^2 \rangle = \frac{g_e^4}{q^4} L^{\mu\nu}_{electron} K_{\mu\nu}(X)$$

$K_{\mu\nu}$ is now some unknown quantity describing $\gamma + P \rightarrow X$

Inelastic Electron-Proton Scattering

The scattering cross section is determined by the Golden Rule for scattering

$$d\sigma = \frac{\hbar^2 \langle |\mathcal{M}|^2 \rangle}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \prod_{j=3}^{n} \frac{1}{2\sqrt{\boldsymbol{p}_j^2 + m_j^2 c^2}} \frac{d^3 \boldsymbol{p}_j}{(2\pi)^3}$$
$$\times \quad (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - p_4 - \cdots - p_n)$$

It is not feasible to measure every single piece of hadronic fragments and to compute a cross section for each possible set of final state particles. We generally only have experimental access to the final momentum of the scattered electron.
This means we measure the inclusive cross section and theoretically calculate it by summing over "all" accessible final states of X and integrating over "all" possible momenta $\boldsymbol{p}_4, \boldsymbol{p}_5 \cdots \boldsymbol{p}_n$,

Inelastic Electron-Proton Scattering

The inclusive cross section is

$$d\sigma = \frac{\hbar^2 g_e^4 L^{\mu\nu}}{4q^4\sqrt{(p_1\cdot p_2)^2-(m_1 m_2 c^2)^2}} \frac{1}{2\sqrt{\boldsymbol{p}_3{}^2+m_3^2c^2}} \frac{d^3\boldsymbol{p}_3}{(2\pi)^3} 4\pi M W_{\mu\nu}$$

where

$$W_{\mu\nu} = \frac{1}{4\pi M}\sum_X \int\cdots\int K_{\mu\nu}(X) \frac{1}{2\sqrt{\boldsymbol{p}_4{}^2+m_4^2c^2}} \frac{d^3\boldsymbol{p}_4}{(2\pi)^3}\cdots$$

$$\frac{1}{2\sqrt{\boldsymbol{p}_n{}^2+m_n^2c^2}} \frac{d^3\boldsymbol{p}_n}{(2\pi)^3} \quad (2\pi)^4\delta^4(q+p-p_4-\cdots-p_n)$$

For a massless electron of energy E striking a proton of mass M we have

$$\sqrt{(p_1\cdot p_2)^2-(m_1 m_2 c^2)^2} = ME$$

Inelastic Electron-Proton Scattering

The outgoing electron has energy E' and

$$\begin{aligned}\frac{d^3p_3}{E_3} &= \frac{|p_3|^2 d|p_3| d\Omega}{E'} \\ &= E' dE' d\Omega\end{aligned}$$

Substituting, the differential cross section becomes

$$\frac{d\sigma}{dE' d\Omega} = \left(\frac{\alpha\hbar}{cq^2}\right)^2 \frac{E'}{E} L^{\mu\nu} W_{\mu\nu}$$

Note that this time E' is not determined kinematically by θ and E since the particles which make up X can soak-up varying amounts of energy. In other words, the total momentum

$$p_{tot} = p_4 + p_5 + \cdots + p_n$$

is not constrained by $p_{tot}^2 = M^2 c^2$

Inelastic Electron-Proton Scattering

So, this time we have two independent variables:

$$\begin{aligned} \text{Experimentalist:} \quad & E', \theta \\ \text{Theorist:} \quad & q^2, x \text{ where } x \equiv -\frac{q^2}{2q \cdot p} \end{aligned}$$

Many other choices exist, this is just convention.

Inelastic Electron-Proton Scattering

From here proceed just as in the elastic scattering case

- Write the most general tensor $W^{\mu\nu}$ which depends on q, p
- Constrain it with the Ward Identity $q_\mu W^{\mu\nu} = 0$
- Write down an expression for the differential cross section in terms of two structure functions $W_1(q^2, x)$ and $W_2(q^2, x)$:

$$\frac{d\sigma}{dE' d\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2(\theta/2)}\right)^2 [2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2)]$$

The structure functions are the generalization of the elastic form factors $K_1(q^2)$ and $K_2(q^2)$.

Summary of Elastic and Inelastic e-P Scattering

- Electron-Proton scattering experiments provide us with a great deal of information about the structure of the proton.
- For elastic scattering we can write the cross section in terms of 2 form factors: $K_1(q^2)$ and $K_2(q^2)$
- For inelastic scattering we can write an inclusive cross section in terms of 2 structure functions: $W_1(q^2, x)$ and $W_2(q^2, x)$

Elastic and Inelastic e-P Scattering

Elastic scattering can be viewed as a special case of inelastic scattering. It is inelastic scattering with the constraint $p_{tot}^2 = M^2c^2$ on the outgoing hadrons. So, it is possible to go from

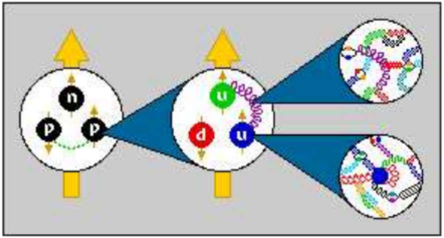
$$\frac{d\sigma}{dE' d\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2(\theta/2)}\right)^2 \left[2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2)\right]$$

to the Rosenbluth formula just by a clever choice of W_1 and W_2. This will do it:

$$W_{1,2}(q^2, x) = -\frac{K_{1,2}(q^2)}{2Mq^2}\delta(x-1)$$

Extending the Rutherford Experiment

Recall that based on a surprisingly high number of large-angle events in elastic α Au scattering, Rutherford deduced atomic substructure (i.e., the nucleus)



In a similar fashion, one can investigate the angles involved in $e\ p$ scattering, particularly in the *deep inelastic scattering* regime where q^2 is large.

The proton was found to have substructure (SLAC, late 1960s). These constituents came to be known as *partons*. Although we now recognize them as quarks and gluons

Deep Inelastic Scattering

So, we have looked briefly at elastic e-p scattering and inelastic e-p scattering, but now we need to consider deep inelastic scattering.

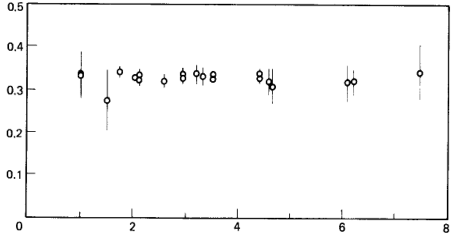
- A low-energy electron scatters elastically off of a proton. This is relatively simple to understand using elastic form factors $K_1(q^2)$ and $K_2(q^2)$
- A medium energy electron scatters inelastically off of a proton. The behaviour is quite complicated and involves inelastic structure functions $W_1(q^2, x)$ and $W_2(q^2, x)$
- A high-energy electron scatters elastically off of partons within the proton. This behaviour is simpler to understand than inelastic scattering and involves parton distribution functions.

Bjorken predicted that at very high energy the q^2 dependence of the inelastic structure functions would go away:

$$\begin{aligned} MW_1(q^2, x) &\rightarrow F_1(x) \\ \frac{-q^2}{2Mc^2 x} W_2(q^2, x) &\rightarrow F_2(x) \end{aligned}$$

This is the deep inelastic scattering regime where q^2 and $q \cdot p$ are both large, but their ratio is not (remember the definition of x).

A plot of F_2 vs q^2:



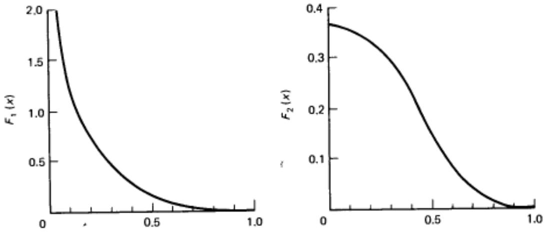
Callan-Gross Relation

Bjorken scaling is a consequence of having partons within the proton, it does not restrict the specific type or parton. ie. these may be partons but not quarks.

In 1969 Callan and Gross suggested that Bjorken's scaling functions are related. They assumed a spin of the partons and found:

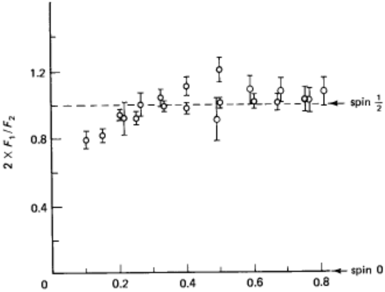
$$Spin - 0partons: \quad \frac{2xF_1(x)}{F_2(x)} = 0$$
$$Spin - 1/2partons: \quad \frac{2xF_1(x)}{F_2(x)} = 1$$

The experimental confirmation of Bjorken scaling and the Callan-Gross relation provided the first compelling evidence for quarks. These relations arise from the fact that at high energies the photon (transferring q^2) interacts with an essentially free quark.


2.0
1.5
1.0
0.5
$F_1(x)$
0
0.5
1.0
0.4
0.3
0.2
0.1
$F_2(x)$
0
0.5
1.0

(a)

(b)


1.2
0.8
0.4
$2 \times F_1/F_2$
spin $\frac{1}{2}$
spin 0
0
0.2
0.4
0.6
0.8

The Parton Model

So, we have experimental proof that quarks exist and that we can have scattering interactions between our beam of electrons and free quarks in the proton.

This means we can treat our interaction $e + q \rightarrow e + q$ as if it were electron-muon scattering (simple Feynman diagram).

Then we can compare the cross section formula to our expression in terms of structure functions and get the form for W_1 and W_2

$$W_1^i = \frac{Q_i^2}{2m_i}\delta(x_i - 1), \; W_2^i = \frac{2m_i c^2 Q_i^2}{q^2}\delta(x_i - 1)$$

where m_i is the mass of the quark, Q_i is its charge, p_i is its momentum and

$$x_i = -\frac{q^2}{2q \cdot p_i}$$

The Parton Model

Of course, we have a fundamental problem. When we collide an electron with a quark (inside a proton) we may know the momentum of the proton but we cannot know the momentum of the quark. Let's suppose that z_i is the fraction of the proton momentum carried by the quark

$$p_i = z_i p$$

This is actually a bit odd in that it assumes that the quark carries the same fraction of p_x, p_y and p_z, for example. So, this model does not allow quarks within protons to move around and quark mass is variable. But let's ignore those minor flaws in our assumptions. Our assumptions imply

$$x_i = \frac{x}{z_i}$$

The Parton Model

We can now write the structure functions like

$$W_1^i = \frac{Q_i^2}{2M}\delta(x - z_i),\ W_2^i = -\frac{2x^2Mc^2}{q^2}Q_i^2\delta(x - z_i)$$

Now we let $f_i(z_i)$ be the probability that the i^{th} quark carries momentum fraction z_i. Integrate over z_i and sum over quarks

$$\begin{aligned} W_1 &= \sum_i \int_0^1 \frac{Q_i^2}{2M}\delta(x - z_i) f_i(z_i) dz_i = \frac{1}{2M}\sum_i Q_i^2 f_i(x) \\ W_2 &= \sum_i \int_0^1 \left(-\frac{2x^2Mc^2}{q^2}\right) Q_i^2\delta(x - z_i) f_i(z_i) dz_i = -\frac{2Mc^2}{q^2}x^2\sum_i Q_i^2 f_i(x) \end{aligned}$$

From these expressions we get

$$\begin{aligned} MW_1 &= \frac{1}{2}\sum_i Q_i^2 f_i(x) \equiv F_1(x) \\ -\frac{q^2}{2Mc^2x}W_2 &= x\sum_i Q_i^2 f_i(x) \equiv F_2(x) \end{aligned}$$

The Parton Model

We have confirmed the Bjorken scaling law and Callan-Gross relation! Further we could show that the differential cross section depends now on only one structure function

$$\frac{d\sigma}{dE'd\Omega} = \frac{F_1(x)}{2M}\left(\frac{\alpha\hbar}{E\sin(\theta/2)}\right)^2\left[1 + \frac{2EE'}{(E-E')^2}\cos^2(\theta/2)\right]$$

So now we just have to go out and measure the "probability distribution functions (PDFs), $f_i(x)$" and use

$$F_1(x) = \frac{1}{2}\sum_i Q_i^2 f_i(x)$$

to predict the cross section.

Quark Distribution Functions

The model we are working with has

$$p_i = z_i p$$

which in turn implies

$$m_i = z_i M$$

So, the momentum fraction carried by the *i*th quark is proportional to its mass.

$$p_i = \frac{m_i}{M} p$$

This makes the probability density a simple delta function

$$f_i(z_i) = \delta\left(\frac{m_i}{M} - z_i\right)$$

Quark Distribution Functions

If a proton is made up of two up quarks and a down quark we then have

$$F_1(x) = \frac{1}{2}\left\{2\left(\frac{2}{3}\right)^2 \delta\left(\frac{m_u}{M} - x\right) + \left(\frac{-1}{3}\right)^2 \delta\left(\frac{m_d}{M} - x\right)\right\}$$

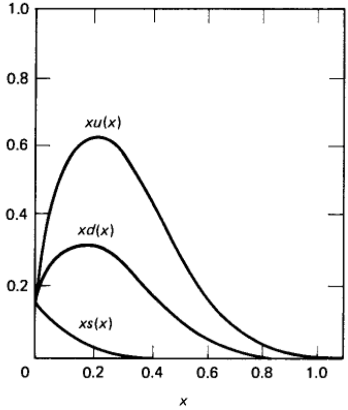
If $m_u = m_d$ we have

$$F_1(x) = \frac{1}{2}\delta\left(\frac{m_u}{M} - x\right),\; F_2(x) = x\delta\left(\frac{m_u}{M} - x\right)$$

I wish it were this simple. However, quarks are actually bound together (ie. we have forgotten gluons) and surrounded by a sea of virtual quarks. The mass of a quark within a hadron is not even a well-defined quantity.

Quark Distribution Functions

Let's say that $u(x)$ is the probability density that the momentum fraction x is carried by a u quark (similarly for $d(x)$). It is tempting to think that, for a proton, $u(x) = 2d(x)$. What does the data say?



Quark Distribution Functions

OK, but the average momentum carried by up quarks is really double the average carried by down quarks. This condition is expressed as a sum rule

$$\int_0^1 xu(x)dx = 2\int_0^1 xd(x)dx$$

(OK, this actually talks about total, not average momentum, but you get the idea.)
A surprise is that both sides of the above equation are measured to be 0.36!

$$\int_0^1 xu(x)dx = 0.36 \quad and \quad \int_0^1 xd(x)dx = 0.18$$

Only 54% of the momentum of a proton is carried by the “valence” (u &d) quarks!

From Halzen and Martin:

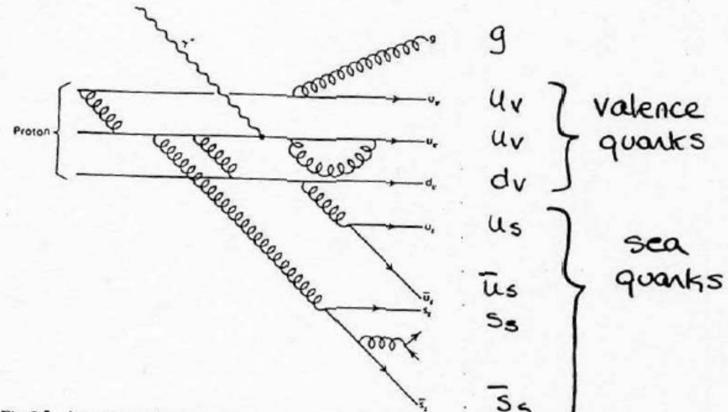


Fig. 9.5 A proton made up of valence quarks, gluons, and slow debris consisting of quark–antiquark pairs.

Quark Distribution Functions

The gluons carry momentum but are uncharged and so do not contribute to this scattering process. Finding missing momentum that does not contribute to scattering signals the presence of uncharged partons....more evidence that gluons actually exist.

Also, there were so-called sea quarks and valence quarks in that picture. We call the original uud the valence quarks and the rest is referred to as the “sea”. The thing about the “sea” is that it does contain charged partons - ie. the electrons can couple to members of the sea and the cross section is different than if there is no sea at all.

OK, if the sea can contribute to the cross section for scattering then it should be included in our parton distribution functions:

$$F_1(x) = \frac{1}{2}\left\{\left(\frac{2}{3}\right)^2 [u(x) + \bar{u}(x)] + \left(\frac{1}{3}\right)^2 [d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]\right\}$$

Now if we want to describe this scattering process we need to measure 6 parton density functions (ignoring c,b,t quarks) and if we really want to describe the full structure of a proton we also need a gluon parton density function. This is getting complicated.

Quark Distribution Functions

Let's clean this up with a little thinking about the sea. It is reasonable to assume that

$$\bar{u}(x) \simeq \bar{d} \simeq \bar{s} \simeq s(x)$$

since the masses of all of these particles are approximately the same. We can also try splitting the $u(x)$ and $d(x)$ into valence and sea components

$$\begin{aligned} u(x) &= u_v(x) + s(x) \\ d(x) &= d_v(x) + s(x) \end{aligned}$$

(presumably s=strange quark = sea) This reduces the problem to 3 unknown functions

$$F_1(x) = \frac{1}{18}\{4u_v(x) + d_v(x) + 12s(x)\}$$

The neutron PDFs are related to the proton PDFs due by isospin, so there are many experiments which can measure these contributions.

Proton and Neutral PDFs

The proton and neutron structure functions (uud and udd):

$$\begin{aligned}
\frac{F_2^p(x)}{x} &= \frac{4}{9}[u^p + \bar{u}^p] + \frac{1}{9}[d^p + \bar{d}^p] + \frac{1}{9}[s^p + \bar{s}^p] \\
\frac{F_2^n(x)}{x} &= \frac{4}{9}[u^n + \bar{u}^n] + \frac{1}{9}[d^n + \bar{d}^n] + \frac{1}{9}[s^n + \bar{s}^n]
\end{aligned}$$

Using isospin invariance

$$\begin{aligned}
u^p &= d^n = u(x) \\
d^p &= u^n = d(x) \\
s^p &= s^n = s(x)
\end{aligned}$$

and

$$\begin{aligned}
\frac{F_2^p(x)}{x} &= \frac{4}{9}[u + \bar{u}] + \frac{1}{9}[d + \bar{d} + s + \bar{s}] \\
\frac{F_2^n(x)}{x} &= \frac{4}{9}[d + \bar{d}] + \frac{1}{9}[u + \bar{u} + s + \bar{s}]
\end{aligned}$$

Proton and Neutral PDFs

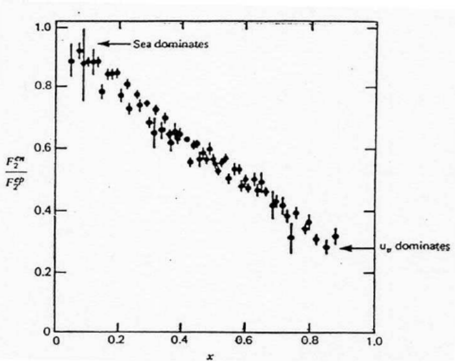
We also know that the quark distributions must give the right quantum numbers

$$\begin{aligned} \int [u - \bar{u}] dx &= 2 \\ \int [d - \bar{d}] dx &= 1 \\ \int [s - \bar{s}] dx &= 0 \end{aligned}$$

So,

$$\begin{aligned} \frac{F_2^p(x)}{x} &= \frac{1}{9}[4u_v + d_v] + \frac{4}{3}s \\ \frac{F_2^n(x)}{x} &= \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}s \end{aligned}$$

Comparison to Experiment



$$\frac{F_2^p(x)}{x} = \frac{1}{9}[4u_v + d_v] + \frac{4}{3}s$$
$$\frac{F_2^n(x)}{x} = \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}s$$

The ratio F^n/F^p tends to 1 if s dominates, tends to 4 if d_v dominates and tends to 1/4 if u_v dominates.

What Does F_2 Look Like Anyway?

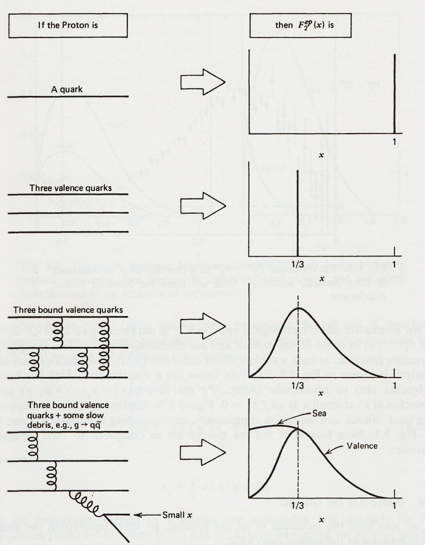
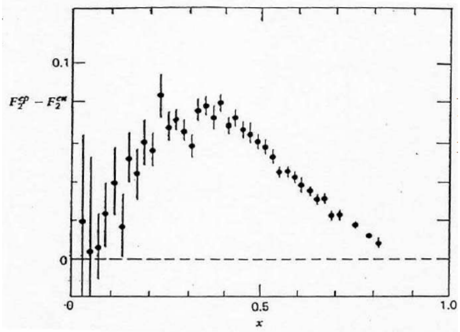


Fig. 9.7 The structure function pictured corresponding to different compositions assumed for the proton.

Another Comparison to Data



$$\frac{1}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3}[u_v(x) - d_v(x)]$$

I expect a fat peak around 1/3.

Scaling Violations

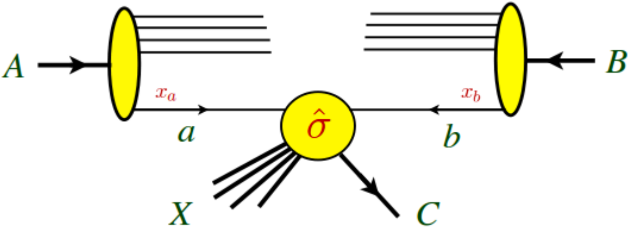
- At extremely large values of q^2, it is observed that the PDFs do depend slightly on q^2 ($F_2(x) \to F_2(x, q^2)$), in conflict with Bjorken's scaling hypothesis.
- In particular, as $|q^2|$ increases, the PDFs decrease at large x and increase at small x. In other words, the closer we look, the more *soft* partons we see.
- Quantitatively, these scaling violations fall within the realm of perturbative QCD. The DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) equations describe the evolution of the PDFs with q^2.

Parton Model Summary

- Data from e-p scattering told us that quarks and gluons exist and continue to tell us the detailed structure of the proton.
- DIS is high energy scattering of electrons off of quarks inside the proton.
- The parton model tells us that the proton is composed of quarks and gluons.
- Quarks are either valence quarks or sea quarks. They are important in different regions of x.
- The quarks only carry about 50% of the momentum of the proton, the other half is carried by gluons. However, the gluons do not participate in the EM interaction and so are left-out of the cross section calculations for e-p scattering.
- This stuff is vitally important at the LHC! We are colliding protons with other protons. How often do we expect g-g collisions vs. q-q collisions at that energy? We need precise measurements of all of the PDFs over a large range of x.

An aside - Parton Distribution Functions

- Inclusive particle production $A\,B \to C\,X$



$$\sigma_{AB\to CX}(p_A, p_B) = \sum_{a,b} \int dx_a\, dx_b\, f_{a/A}(x_a, \mu)\, f_{b/B}(x_b, \mu)$$
$$\times \sum_n \alpha_s^n(\mu)\, \hat{\sigma}^{(n)}_{ab\to CX}(x_a p_A, x_b p_B, Q/\mu)$$

→ universal functions $f_{a/A}$ characterize internal structure of bound state A

Parton distributions in nucleons

- PDFs extracted in global QCD analyses of data from deep-inelastic l-h scattering; lepton-pair, weak boson & jet production in h-h scattering, ...

				χ^2		
	Experiment	Ref.	# points	CJ12min	CJ12mid	CJ12max
DIS F_2	BCDMS (p)	[13]	351	434	436	437
	BCDMS (d)	[13]	254	294	297	302
	NMC (p)	[14]	275	434	432	430
	NMC (d/p)	[15]	189	179	177	182
	SLAC (p)	[16]	565	456	455	456
	SLAC (d)	[16]	582	394	388	396
	JLab (p)	[17]	136	170	169	170
	JLab (d)	[17]	136	124	125	126
DIS σ	HERA (NC e^-)	[18]	145	117	117	118
	HERA (NC e^+)	[18]	384	595	596	596
	HERA (CC e^-)	[18]	34	19	19	19
	HERA (CC e^+)	[18]	34	32	32	32
Drell-Yan	E866 (p)	[19]	184	220	221	221
	E866 (d)	[19]	191	297	307	306
W asymmetry	CDF 1998 (ℓ)	[20]	11	14	16	18
	CDF 2005 (ℓ)	[21]	11	11	11	10
	DØ 2008 (ℓ)	[22]	10	4	4	4
	DØ 2008 (e)	[23]	12	40	36	34
	CDF 2009 (W)	[24]	13	20	25	41
Z rapidity	CDF (Z)	[25]	28	29	27	27
	DØ (Z)	[26]	28	16	16	16
jet	CDF run 1	[27]	33	52	52	52
	CDF run 2	[28]	72	14	14	14
	DØ run 1	[29]	90	21	20	19
	DØ run 2	[30]	90	19	19	20
γ+jet	DØ 1	[31]	16	6	6	6
	DØ 2	[31]	16	13	13	12
	DØ 3	[31]	12	17	17	17
	DØ 4	[31]	12	17	16	17
	TOTAL		3958	4059	4055	4096
	TOTAL + norm			4075	4074	4117

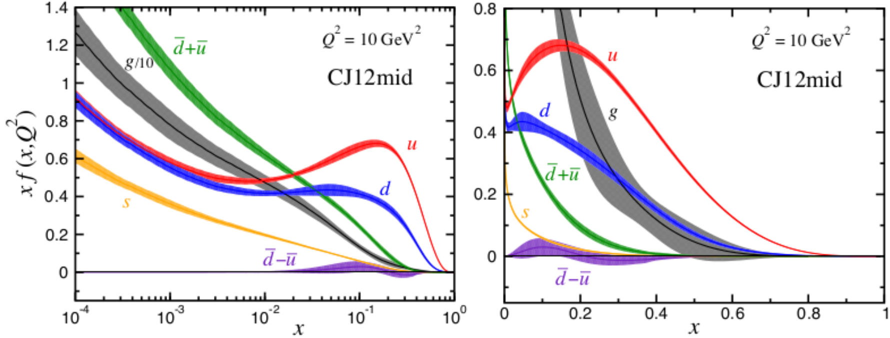
~ 4,000 spin-averaged data points over large range of x and Q^2

Parton distributions in nucleons

- Several groups dedicated to global PDF analysis
  - CTEQ (Coordinate Theoretical-Experimental Project on QCD)
    - CT (CTEQ-Tung et al.) LHC focus
    - CJ (CTEQ-JLab) includes high x, low Q^2
    - nCTEQ nuclear PDFs
  - MSTW (Martin-Stirling-Thorne-Watt) LHC focus, strong data cuts
  - ABM (Alekhin-Bluemlein-Moch) LHC focus, some lower Q^2
  - HERAPDF only H1 & ZEUS data
  - JR (Jimenez-Delgado–Reya) dynamically generated from low Q^2
  - NNPDF "neural networks", strong data cuts

Parton distributions in nucleons

- Example of recent PDFs, from CJ12 analysis



Owens, Accardi, WM
PRD 87, 094012 (2013)

The Feynman Rules for Chromodynamics

Before stating the Feynman rules for QCD, lets look at:

- strong coupling constant
- the quarks represented in “color” space
- color state of gluons
- Gell-Mann “λ matrices” and their commutators

Coupling constant of QCD

Electromagnetic force

The strength of the electromagnetic force is set by the coupling constant

$$g_e = e\sqrt{4\pi/\hbar c} = \sqrt{4\pi\alpha}$$

where g_e is related to the charge of the positron or the fine structure constant.

Chromodynamic force

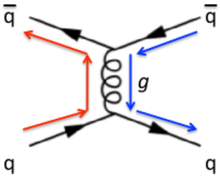
The strength of the chromodynamic force is set by the “strong” coupling constant

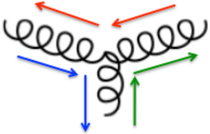
$$g_s = \sqrt{4\pi\alpha_s}$$

where g_s is related to the fundamental unit of color.

Color

Quarks come in three colors, red (r), blue (b), and green (g).





Each gluon carries one unit of color and one of anticolor.
The quark color changes at a quark-gluon vertex, and the difference is carried off by the gluon.

Because the gluons themselves carry color (in contrast to the photon, which is electrically neutral), they couple directly to one another.

There is a "three-gluon" vertex and a "four-gluon" vertex.

Color Symmetry & Quark states in QCD

- Strong coupling is same for all color states
- Each of the three colors is conserved seperately
- A quark state in QCD requires not only the Dirac spinor $u^{(s)}(p)$, giving its momentum and spin, but also a three-element column vector c, giving its color:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Gluon Color states

Since each gluon carries one unit of color and one of anticolor. Naively there should be nine states:

$$r\bar{r}, r\bar{b}, r\bar{g}, b\bar{r}, b\bar{b}, b\bar{g}, g\bar{r}, g\bar{b}, g\bar{g}$$

In terms of color $SU(3)$ symmetry these nine states constitute:

- a 'color octet' (allowed for gluons)

$$\left[\begin{array}{ll} |1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} & |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ |2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} & |6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2} \\ |3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} & |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ |4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2} & |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \end{array}\right]$$

- a 'color singlet' (forbidden for gluons)

$$|9\rangle = (r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$$

Confinement requires that all naturally occurring particles be color singlets and appear as free particles.

Gluon states in QCD

Like the photon, gluons are massless particles of spin 1; they are represented by a polarization vector, ϵ^μ , which is orthogonal to the gluon momentum, p:

- Lorentz Condition: $\epsilon^\mu p_\mu = 0$
- Coulomb Gauge: $\epsilon^0 = 0$, $\boldsymbol{\epsilon} \cdot \boldsymbol{p} = 0$

To describe the color state of the gluon, we need in addition an eight-element column vector, a:

$$a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ for } |1\rangle, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ for } |7\rangle, \quad \text{and so on}$$

SU(3) matrices

- Color symmetry is described by $SU(3)$ group. The generators of $SU(3)$ are eight $3X3$ matrices λ^a. Gell-Mann "λ matrices":

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- The commutators of λ matrices define the "structure constants", $f^{\alpha\beta\gamma}$, of $SU(3)$ group:

$$[\lambda^\alpha, \lambda^\beta] = 2\, i\, f^{\alpha\beta\gamma} \lambda^\gamma$$

(summation over γ from 1 to 8 is implied by repeated indices)

The Feynman Rules for QCD

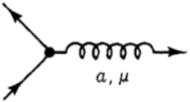
1. Draw the Feynman diagrams with the minimum number of vertices.
2. Label the momenta (p_1, p_2, etc.) but also add spin and color labels and add q's to the internal lines. Indicate on the diagram the indices (space-time and color) for each gluon.
3. External lines contribute:

Quark
- incoming ( ): $u^{(s)}(p)c$
- outgoing ( ): $\bar{u}^{(s)}(p)c^{\dagger}$

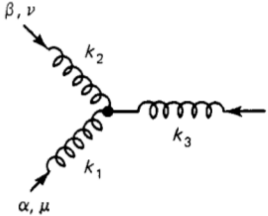
Antiquark
- incoming ( ): $\bar{v}^{(s)}(p)c^{\dagger}$
- outgoing ( ): $v^{(s)}(p)c$

Gluon
- incoming (μ): $\epsilon_{\mu}(p)a^{\alpha}$
- outgoing (α, μ): $\epsilon^{*}_{\mu}(p)a^{\alpha *}$

4. Each vertex contributes a factor:

Quark-gluon (

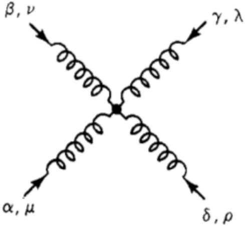
): $\dfrac{-ig_s}{2}\lambda^\alpha\gamma^\mu$

Three gluon (

):

$$-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(k_1 - k_2)_\lambda + g_{\nu\lambda}(k_2 - k_3)_\mu + g_{\lambda\mu}(k_3 - k_1)_\nu]$$

Gluon momenta (k_1, k_2, k_3) are assumed to point into the vertex; change their signs if any point outward in the diagram.

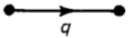
Four gluon ():

$$-ig_s^2[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho}) + f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})]$$

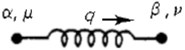
Summation over η implied.

Feynman Rules

5. Each internal line contributes a factor:

Quark-antiquark (

): $\dfrac{i(\not{q} + mc)}{q^2 - m^2c^2}$

Gluon (

): $\dfrac{-ig_{\mu\nu}\,\delta^{\alpha\beta}}{q^2}$

6. Conservation of energy and momentum makes each vertex contribute

$$(2\pi)^4\delta^4(k_1 + k_2 + k_3)$$

particles coming in are positive and going out are negative (antiparticles are opposite).

Feynman Rules

7. Integrate over internal momenta. For each internal momentum q write a factor

$$\frac{d^4q}{(2\pi)^4}$$

and integrate.

8. Cancel the delta function. Erase the term that looks like

$$(2\pi)^4\delta^4(p_1 + p_2 + \cdots - p_n)$$

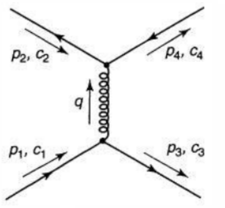
Multiply by i, what remains is $\mathcal{M}$

Color factors

- Consider the interaction between two quarks (also a quark and an antiquark) in lowest-order QCD.
- Cannot observe quark-quark scattering directly in the laboratory (although hadron-hadron scattering is an indirect manifestation), so won't be looking for cross sections here.
- Concentrate on the effective potetials between quarks - the QCD analog of the Coulomb potential in electrodynamics.

Quark-antiquark scattering

Consider the interaction of a quark and an antiquark in QCD assuming that they have different flavors.



Applying the Feynman Rules, the amplitude is given by:

$$-i\mathcal{M} = [\bar{u}(3)c_3^\dagger]\left[-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu\right][u(1)c_1]\left[\frac{-ig_{\mu\nu}\,\delta^{\alpha\beta}}{q^2}\right]$$

$$\times[\bar{v}(2)c_2^\dagger]\left[-i\frac{g_s}{2}\lambda^\beta\gamma^\nu\right][v(4)c_4]$$

$$\mathcal{M} = \frac{-g_s^2}{4}\frac{1}{q^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{v}(2)\gamma_\mu v(4)](c_3^\dagger\lambda^\alpha c_1)(c_2^\dagger\lambda^\alpha c_4)$$

Quark-antiquark scattering

The amplitude is same as electron-positron scattering except that: g_e is replaced with g_s and additional "color factor":

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$$

The potential describing the $q\bar{q}$ interaction is the same as that acting in electrodynamics between two opposite charges only with α replaced by $f\ \alpha_s$

$$V_{q\bar{q}}(r) = -f\,\frac{(\alpha_s \hbar c)}{r}$$

The color factor depends on the color state of the interacting quarks. From a quark and an antiquark we can make a "color singlet" and a "color octet"

Calculating the color factors (see the text for details) for color singlet and color octet states, the potential describing the $q\bar{q}$ interaction is:

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{(\alpha_s \hbar c)}{r} \qquad \text{(color singlet)}$$

$$V_{q\bar{q}}(r) = \frac{1}{6}\frac{(\alpha_s \hbar c)}{\mathrm{r}} \qquad \text{(color octet)}$$

From the signs see that the force is attractive in the color singlet but repulsive for the octet.
This explains why quark-antiquark binding (to form mesons) occurs in the singlet configuration but not in the octet (which would have produced colored mesons).